STAT 107 – Spring 2021

Confidence Interval for Population Mean

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***σ* is known:**

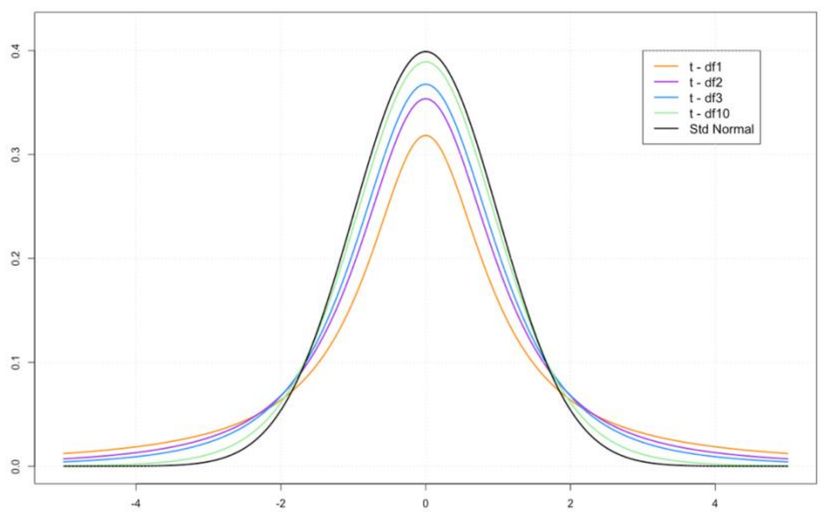
A confidence interval for the population mean μ with confidence level $(1-\alpha)100\%$:

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

***σ* is <u>un</u>known:**

A confidence interval for the population mean μ With confidence level $(1-\alpha)100\%$:

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



Rules for computing CIs:

<u>Case 1</u>: σ is known

- Normal population

- Unknown population

<u>Case 2</u>**:** σ is unknown

- Normal population

- Unknown population

Example 1:

Suppose the lifetime of a particular brand of light bulbs is normally distributed with standard deviation of $\sigma = 75$ hours and unknown mean.

a) Suppose the sample average lifetime of $n = 49$ bulbs is $\bar{x} = 843$ hours. Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

b) Construct a 90% confidence interval for the overall average lifetime for light bulbs of this brand.

c) Construct a 99% confidence lower bound for the overall average lifetime for light bulbs of this brand.

Lower Bound:

A $(1-\alpha)100\%$ lower bound for μ:

$$\bar{x} - z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} - t_\alpha \cdot \frac{s}{\sqrt{n}}$$

Upper Bound:

A $(1-\alpha)100\%$ upper bound for μ:

$$\bar{x} + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} + t_\alpha \cdot \frac{s}{\sqrt{n}}$$

Example 2:

A manufacturer of TV sets wants to find the average selling price of a particular model. A random sample of 25 different stores gives the mean selling price as $342 with a sample standard deviation of $14. Assume the prices are normally distributed.
Construct a 95% confidence interval for the mean selling price of the TV model.

Sample Size Calculation

The minimum required sample size in estimating the population mean μ to within ε with $(1-\alpha)100\%$ confidence is

$$n = \left[\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon}\right]^2$$

Always round n up!

Example 3:

How many test runs of an automobile are required for determining its average miles-per-gallon rating on the highway to within 0.5 miles per gallon with 95% confidence, if a guess is that the variance of the population of miles per gallon is about 6.25 miles?