STAT 107 – Fall 2020

Bayes' Theorem

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Law of Total Probability

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(B) \cdot P(A \mid B) + P(B') \cdot P(A \mid B') \end{aligned}$$

In general,

$$P(A) = \sum_{i=1}^{m} P(B_i) \cdot P(A \mid B_i)$$

Example 1:

Based on past data, it is believed that the proportion of individuals who have COVID-19 in Champaign-Urbana area is 0.025. A test for COVID-19 is positive in 94% of the people who have the virus and in 4% of the people who do not.

a) Find the probability of receiving a positive result from this COVID-19 test.

b) If a person received a positive result from this test, what is the probability that he/she has the virus?

Bayes' Theorem:

$$P(B \mid A) = \frac{P(B) \cdot P(A \mid B)}{P(B) \cdot P(A \mid B) + P(B') \cdot P(A \mid B')}$$

c) If a person received a negative result from this test, what is the probability that he/she actually doesn't have the virus?

Example 2:

Company A is developing a new pregnancy test. Based on their experiments, the test gives the correct result about 88% of the time, meaning the test result is positive when the person is pregnant, it is negative when the person is not pregnant. Suppose that 55% of the women who take the test are pregnant.

What is the probability that the person is actually pregnant when their test result is negative?