STAT 107 – Fall 2020

Conditional Probability

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The conditional probability of A, given B (the probability of event A, computed on the assumption that event B has happened) is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad \text{P(B)} \neq 0$$

Similarly, the **conditional probability of B, given A** is

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \qquad \text{P(A)} \neq 0$$

Example 1:

A study is conducted on a group of 48 STAT 430 students. Each student is asked whether he/she lives in the Champaign-Urbana area for Fall 2020. The results are recorded in the following table:

	Live in CU	**Not live in CU**	**Totals**
Male	13	11	24
Female	5	19	24
Totals	18	30	48

a) What is the probability that a student is female, given that he/she lives in CU?

b) Suppose a student is female. What is the probability that she does not live in CU?

Example 2:

The probability that Ha orders food delivery on a random day is 0.45. Also note that the probability that Ha does not leave the house on a random day is 0.8. The probability Ha both orders food delivery and does not leave the house is 0.35.

a) What is the probability of Ha does not leave the house given that you saw her ordered food delivery?

b) Suppose you saw Ha at Café Kopi (meaning she left the house), what is the probability that she will not order food delivery?

Multiplication Law of Probability

If A and B are any two events, then

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

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Example 3:

It is known that 30% of all the students at Anytown College live off campus. Suppose also that 48% of all the students are females. Of the female students, 25% live off campus.

a) What is the probability that a randomly selected student is female and lives off campus?

b) What is the probability that a randomly selected student either is a female or lives off campus, or both?