Events A and B are independent if and only if

\[ P(B \mid A) = P(B) \quad \text{or} \quad P(A \mid B) = P(A) \]
\[ \text{or} \quad P(A \cap B) = P(A) \cdot P(B) \]

**Example 1:**

The probability that Ha orders food delivery on a random day is 0.45. Also note that the probability that Ha does not leave the house on a random day is 0.8. The probability Ha both orders food delivery and does not leave the house is 0.35.

Are the events \{Ha orders food delivery\} and \{Ha does not leave the house\} independent?

**Example 2:**

Suppose Ha, Dave, Danny and Katie play cornhole for fun. Suppose Ha hits the target with probability 0.2 (she is really bad at sports), Dave misses only 10% of the time. Danny and Katie hit the target half of the time each. Assume their attempts are independent of each other.

a) Find the probability that all of them will hit the target.

b) What is the probability that exactly one of them will hit the target?
c) What is the probability that at least one of the m will hit the target?

**Idea:** $P(\text{at least one of } A_i \text{ occurs} ) = 1 - P(\text{none of } A_i \text{ occurs} )$

$P( A_1 \text{ or } A_2 \text{ or } A_3 \text{ or ... } \text{ or } A_n ) = 1 - P(\text{(not } A_1) \text{ and } \text{(not } A_2) \text{ and } \text{(not } A_3) \text{ and ... and } \text{(not } A_n) )$

$P( A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n ) = 1 - P( A_1' \cap A_2' \cap A_3' \cap \ldots \cap A_n' )$

For independent events

$P( A_1 \text{ or } A_2 \text{ or } A_3 \text{ or ... } \text{ or } A_n ) = 1 - P(\text{not } A_1) \cdot P(\text{not } A_2) \cdot P(\text{not } A_3) \cdot \ldots \cdot P(\text{not } A_n)$

$P( A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n ) = 1 - P( A_1') \cdot P( A_2') \cdot P( A_3') \cdot \ldots \cdot P( A_n')$

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**Example 3:**

A major oil company has decided to drill independent test wells in the Alaskan wilderness. The probability of any well producing oil is 0.30. Find the probability that the fifth well is the first to produce oil.