



Hypothesis Test for Two Means

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$X_1, X_2, \dots, X_{n_1}$   
from population 1  
mean  $\mu_1$ , std  $\sigma_1$

$Y_1, Y_2, \dots, Y_{n_2}$   
from population 2  
mean  $\mu_2$ , std  $\sigma_2$

When one of these 2 conditions are satisfied:

- $n_1$  and  $n_2$  are large ( $\geq 30$ )
- Population 1 and population 2 are approximately normal

Then,  $(\bar{X} - \bar{Y})$  is approximately normal with mean  $\mu_1 - \mu_2$  and standard deviation  $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ .

And we can perform a hypothesis test for:

$$H_0: \mu_1 - \mu_2 = \delta_0 \quad \text{vs.} \quad \begin{aligned} H_1: \mu_1 - \mu_2 &\neq \delta_0 \\ H_1: \mu_1 - \mu_2 &> \delta_0 \\ H_1: \mu_1 - \mu_2 &< \delta_0 \end{aligned}$$

Test statistic:

$$Z = \frac{(\bar{X} - \bar{Y}) - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

If  $\sigma_1$  and  $\sigma_2$  are unknown AND  $n_1$  and  $n_2$  are large, then approximate  $\sigma_1$  with  $s_1$  and  $\sigma_2$  with  $s_2$ .

**Example 1:**

Dr. Statman claims that his new revolutionary study method “Study While You Sleep” (SWYS) is more effective than the traditional study methods. In an experiment, 250 students enrolled in the same section of STAT 400 at UIUC were divided into 2 groups. 100 students volunteered to study using SWYS method, and the other 150 students did whatever the students usually do. At the end of the semester, the average of the total number of points (out of 500) were compared for the 2 groups.

*Note:* This is not a good experiment design!

	SWYS	Traditional
Average total points	450	410
Standard deviation	20	45

a) Perform the appropriate test at a 1% level of significance.

Step 0: Check the assumptions

Step 1: State the hypotheses

Step 2: Compute the test statistic

Step 3: Compute the p-value

Step 4: State the conclusion

$p\text{-value} \leq \alpha$ : Reject  $H_0$

$p\text{-value} > \alpha$ : Fail to reject  $H_0$

b) After the semester, looking at the data, Dr. Statman makes an even bolder claim! He thinks that the SWYS study method improve students' total number of points over 30 points!  
Conduct a hypothesis test at 5% level of significance to test this claim.

Step 1: State the hypotheses

Step 2: Compute the test statistic

Step 3: Compute the p-value

Step 4: State the conclusion

$p\text{-value} \leq \alpha$ : Reject  $H_0$

$p\text{-value} > \alpha$ : Fail to reject  $H_0$