STAT 107 – Fall 2020

Confidence Interval for Population Proportion

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The sample proportion:

$$\hat{p} = \frac{x}{n}$$

A $(1-\alpha)100\%$ confidence interval for the population mean p:

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Example 1:

Just prior to an important election, in a random sample of 749 voters, 397 preferred Candidate Y over Candidate Z. Construct a 90% confidence interval for the overall proportion of voters who prefer Candidate Y over Candidate Z.

Lower Bound:

A $(1-\alpha)100\%$ lower bound for p:

$$\hat{p} - z_{\alpha} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Upper Bound:

A $(1-\alpha)100\%$ upper bound for p:

$$\hat{p} + z_{\alpha} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

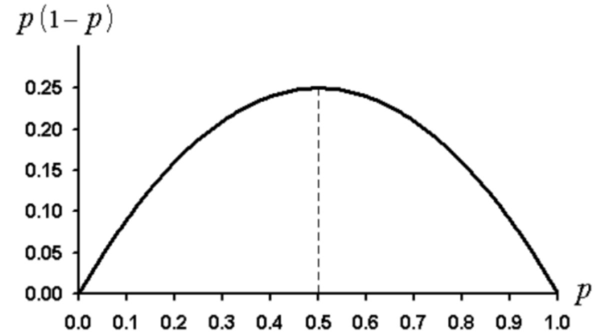
Construct a 95% confidence upper bound for the overall proportion of voters who prefer Candidate Y over Candidate Z.

Sample Size Calculation

The minimum required sample size to obtain a confidence interval for the population proportion p with specified margin of error ε is

$$n = \left(\frac{z_{\alpha/2}}{\varepsilon}\right)^2 p^*(1 - p^*)$$

Always round n up!



Conservative approach:

- If it is possible that $p = 0.5$, use $p^* = 0.5$.
- If it is not possible that $p = 0.5$, use $p^* =$ a possible value of p that is closest to 0.5

Example 3:
A television station wants to estimate the proportion of the viewing audience in its area that watch its evening news. Find the minimum sample size required to estimate that proportion to within 3% with 95% confidence if …

a) no guess as to the value of that proportion is available.

b) it is known that the station's evening news reaches at most 30% of the viewing audience.