Confidence Interval for Population Mean

Ha Khanh Nguyen (hknguyen)

\( \sigma \) is known:
A confidence interval for the population mean \( \mu \) with
confidence level \( (1 - \alpha)100\% \):

\[
\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}
\]

\( \sigma \) is unknown:
A confidence interval for the population mean \( \mu \)
With confidence level \( (1 - \alpha)100\% \):

\[
\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}
\]

Rules for computing CIs:

Case 1: \( \sigma \) is known
- Normal population
- Unknown population

Case 2: \( \sigma \) is unknown
- Normal population
- Unknown population
Example 1:
Suppose the lifetime of a particular brand of light bulbs is normally distributed with standard deviation of \( \sigma = 75 \) hours and unknown mean.

a) Suppose the sample average lifetime of \( n = 49 \) bulbs is \( \bar{x} = 843 \) hours. Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

b) Construct a 90% confidence interval for the overall average lifetime for light bulbs of this brand.

c) Construct a 99% confidence lower bound for the overall average lifetime for light bulbs of this brand.

**Lower Bound:**
A \((1 - \alpha)\)100\% lower bound for \( \mu \):
\[
\bar{x} - z_{\alpha} \cdot \frac{\sigma}{\sqrt{n}}
\]
\[
\bar{x} - t_{\alpha} \cdot \frac{s}{\sqrt{n}}
\]

**Upper Bound:**
A \((1 - \alpha)\)100\% upper bound for \( \mu \):
\[
\bar{x} + z_{\alpha} \cdot \frac{\sigma}{\sqrt{n}}
\]
\[
\bar{x} + t_{\alpha} \cdot \frac{s}{\sqrt{n}}
\]
Example 2:
A manufacturer of TV sets wants to find the average selling price of a particular model. A random sample of 25 different stores gives the mean selling price as $342 with a sample standard deviation of $14. Assume the prices are normally distributed. Construct a 95% confidence interval for the mean selling price of the TV model.

Sample Size Calculation
The minimum required sample size in estimating the population mean \( \mu \) to within \( \varepsilon \) with \( (1 - \alpha)100\% \) confidence is

\[
n = \left[ \frac{Z_{\alpha/2} \cdot \sigma}{\varepsilon} \right]^2
\]

Always round \( n \) up!

Example 3:
How many test runs of an automobile are required for determining its average miles-per-gallon rating on the highway to within 0.5 miles per gallon with 95% confidence, if a guess is that the variance of the population of miles per gallon is about 6.25 miles?

To-do:
- Finish Lab 09, commit and push the lab using git commands!
- Get started with HW 8 on Prairie Learn!