Example 1:
The amount of sulfur in the daily emissions from a power plant has a normal distribution with mean of 134 pounds and a standard deviation of 22 pounds. For a random sample of 5 days, find the probability that the total amount of sulfur emissions will exceed 700 pounds.

\[
X = \text{the amount of sulfur from a power plant in a day}
\]
\[
X \sim N(\mu_X = 134, \sigma_X = 22)
\]
\[
X_1 + X_2 + X_3 + X_4 + X_5 = \sum X \sim N(n\mu_X, n\sigma_X^2) = N(5 \cdot 134, \sqrt{5} \cdot 22)
\]
\[
P(\sum X > 700) = 1 - P(\sum X < 700) \approx 0.27098
\]

Example 2:
The distribution of the baggage weights for passengers using a particular airline has a mean of 20 lbs and a standard deviation of 5 lbs. What is the probability that for (a random sample of) 100 passengers ...

a) the total luggage weight is less than 2100 lbs?

\[
\sum X
\]
\[
X = \text{the baggage weight of a passenger} \quad \mu_X = 20 \text{ lbs} \quad \sigma_X = 5 \text{ lbs}
\]
By CLT, \[
\sum X \sim N(n\mu_X, \sigma_X^2\sqrt{n}) = N(100 \cdot 20, 5 \sqrt{100})
\]
\[
P(\sum X < 2100) \approx 0.97725
\]

b) the average weight of the sample is within 0.5 lbs of the overall mean?

\[
19.5 < \bar{X} < 20.5
\]
\[
\mu_X = 20
\]
\[
P(19.5 < \bar{X} < 20.5) \approx 0.68
\]
By CLT, \[
\bar{X} \sim N(\mu_X, \frac{\sigma_X}{\sqrt{n}}) = N(20, \frac{5}{\sqrt{100}})
\]
\[
0.5
\]
Example 3:
The amount of cereal dispensed into "16-ounce" boxes of Captain Crisp cereal was normally distributed with mean 16.12 ounces and standard deviation 0.20 ounces.

a) What proportion of boxes are "underfilled"? That is, what is the probability that the amount dispensed into a box is less than 16 ounces?

$X = \text{amount dispensed into a box}$

$X \sim N(\mu_X = 16.12, \sigma_X = 0.2)$

$p(X < 16) = 0.274425$

b) Find the probability that the sample mean amount of cereal for a random sample of 9 boxes is less than 16 ounces.

$\bar{X} \sim N(\mu = \mu_X = 16.12, \sigma = \frac{\sigma_X}{\sqrt{n}} = \frac{0.2}{\sqrt{9}})$

$p(\bar{X} < 16) \approx 0.03593$

c) Suppose that the machine can be adjusted to change the mean while the standard deviation remains at 0.20 ounces. What must the mean be so that only 20% of all the boxes are "underfilled"?

Find $k$ such that

$X \sim N(\mu_X = k, \sigma_X = 0.2)$

and $p(X < 16) = 0.2$

$\Rightarrow p\left(\frac{X - \mu_X}{\sigma_X} < \frac{16 - k}{0.2}\right) = 0.2$

$\Rightarrow p\left(Z < \frac{16 - k}{0.2}\right) = 0.2$

Use norm.ppf(), $k = 16.168$

To-do:
- Finish Lab 08, commit and push the lab using git commands!
- Finish HW 7 on Prairie Learn!