Expected Value

The **expected value** of a random variable is the **average** of all outcomes of the trials if we are to repeat the trial many times (think 100,000 or 1 million times!).

\[
\mu_X = E(X) = \sum_{x} x \cdot f(x) = x_1 f(x_1) + x_2 f(x_2) + \ldots
\]

**Example 1:**

Let \( X \) = the number of pizza Alex orders per week.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( x \cdot f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>1.05</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

\[
\mu_X = E(X) = 2.85
\]

Variance

The variance of a random variable is a measurement of how spread out the outcomes of the trial are! (just like variance of data)

\[
\sigma_X^2 = Var(X) = \sum_{x} (x - \mu_X)^2 \cdot f(x)
\]

**Example 1:** (cont.)

b) Find the variance of \( X \).

\[
\sigma_X^2 = Var(X) = 1.6275
\]
Standard Deviation
\[ \sigma_X = SD(X) = \sqrt{Var(X)} \]

Example 1: (cont.)

c) Find the standard deviation of X.
\[
\sigma_X = SD(X) = \sqrt{1.6275} \approx 1.275
\]

Example 2:
We are flipping 3 loaded coins. With these coins, the probability of getting tail is 0.3. Let X be the number of heads we get.

a) What is the probability distribution of X?
\[
S = \{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\}
\]
\[
X = \{0, 1, 2, 3\}
\]
\[
P(TTT) = (0.3)^3 = 0.027
\]
\[
P(TTH) = (0.3)^2 (0.7) = 0.063
\]
\[
P(THT) = (0.3) (0.7)^2 = 0.147
\]
\[
P(THH) = (0.7)^3 = 0.343
\]

b) On average, how many heads will we get each time?
\[
X \quad | \quad f(x) \quad | \quad x \cdot f(x)
\]
\[
\begin{array}{l|l|l}
0 & 0.027 & 0 \\
1 & 3 \cdot 0.063 = 0.189 & 0.189 \\
2 & 3 \cdot 0.147 = 0.441 & 0.882 \\
3 & 0.343 & 1.029 \\
\end{array}
\]
\[\mu_X = E(X) = 2.1\]

c) Find the variance and the standard deviation of X?
\[
X \quad | \quad f(x) \quad | \quad (x - \mu_X)^2 \cdot f(x)
\]
\[
\begin{array}{l|l|l}
0 & 0.027 & 0.11907 \\
1 & 0.189 & 0.22869 \\
2 & 0.441 & 0.00441 \\
3 & 0.343 & 0.27983 \\
\end{array}
\]
\[Var(X) = 0.63\]
\[SD(X) = 0.7987\]