What Does A 95% Confidence Interval Mean?

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\( X = \) the amount of coffee a U of I student drinks per day
\( \mu = \) the true average amount of coffee a U of I student drinks per day
\( \bar{x} = \) sample average for a particular sample

A 95% confidence interval for \( \mu \): \((8.4, 10.2)\) oz

What does confidence level of 95% mean?

\( 1. \) There is a 95% probability that the population mean is between 8.4 oz and 10.2 oz.
\( 2. \) There is a 95% probability that the sample mean is between 8.4 oz and 10.2 oz.
\( 3. \) 95% of the population data are between 8.4 oz and 10.2 oz.
\( 4. \) 95% of the sample data are between 8.4 oz and 10.2 oz.
\( 5. \) If we take many samples of the same size and each compute a 95% CI for \( \mu \), about 95% of the sample means would be between 8.4 oz and 10.2 oz.
\( 6. \) If we take many samples of the same size and each compute a 95% CI for \( \mu \), about 95% of the CIs will contain the true mean \( \mu \).

\[ z\text{-CI: } \bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \]
\[ t\text{-CI: } \bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \]

\[ df = n - 1 \]

\[ a \text{ CI for a particular sample} \]

\[ \mu \]

\[ \bar{x} \]

\[ \text{NOTE: When can we calculate CI for } p? \]

\[ \text{When } np \geq 5 \text{ and } n(1-p) \geq 5! \]